IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Vacuum radiation pressure on moving mirrors

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1994 J. Phys. A: Math. Gen. 27 2167

(http://iopscience.iop.org/0305-4470/27/6/037)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.68
The article was downloaded on 02/06/2010 at 00:16

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. 27 (1994) 2167–2180. Printed in the UK

Vacuum radiation pressure on moving mirrors

P A Maia Neto†

Laboratoire de Spectroscopie Hertzienne de l'ENS‡, case 74, Université Pierre et Marie Curie, 4 Place Jussieu, F75252 Paris Cedex 05, France

Received 16 August 1993, in final form 14 December 1993

Abstract. We consider a perfectly reflecting plane mirror moving in the vacuum of the electromagnetic field. The motional modification of the Maxwell stress tensor is computed up to first order in the displacement of the mirror. The resulting dissipative force is shown to be related to the creation of travelling-wave low-frequency photons and to obey the fluctuation-dissipation theorem.

1. Introduction

Quantum field theory with moving boundaries has been studied since the early 70's. As shown by Fulling and Davies for the particular case of a one-dimensional scalar-field model, a mirror non-uniformly accelerated in vacuum emits radiation [1]. In the non-relativistic limit, the corresponding radiation reaction force was shown [2] to obey the fluctuation–dissipation theorem of linear response theory [3]. The linear approximation (corrections corresponding to the motional effect considered to first order in mirror's displacement) was first employed by Ford and Vilenkin in order to obtain the vacuum radiation pressure for a three-dimensional scalar field [4].

The fluctuations of the electromagnetic radiation pressure on a flat perfectly reflecting mirror at rest in (three-dimensional) vacuum were computed by Barton [5]. The dissipative force was then inferred by using the fluctuation–dissipation theorem [6–8]. This method was recently criticized by Eberlein [9] on the basis that no Hamiltonian formalism exists for perfectly reflecting moving mirrors, as shown in [10] (such approach is possible, however, for dieletric plates [11]). Hence, the standard perturbation theory, which is vital in Kubo's approach to linear response theory [3], cannot be applied to such a model. Furthermore, the dispersive component of the motional force cannot be inferred from the results in [6–8], for they predict a force depending on the instantaneous position of the mirror [12], and thus dispersion relations are not satisfied, as discussed in [8].

In this paper we derive a modification of the Maxwell stress tensor to first order of mirror displacement. We then obtain the motional radiation pressure (in the linear approximation) exerted by the electromagnetic field on an (infinite) perfectly reflecting plane mirror. We find a dissipative component which is in agreement with the fluctuation–dissipation theorem—thus confirming the results in [6–8]—and a divergent dispersive component. The physical origin of both components is clearly identified in our approach. The dispersive force is

† Present adress: Departamento de Física, PUC/Rio, Caixa Postal 38071, 22452 Rio de Janeiro RJ, Brazil.
‡ Unité de l'Ecole Normale Supérieure et de l'Université Pierre et Marie Curie, associée au Centre National de la Recherche Scientifique.

0305-4470/94/062167+14$19.50 © 1994 IOP Publishing Ltd

related to evanescent waves which are confined near the surface of the mirror, whereas the dissipative force is the mechanical effect of a motionally induced emission of radiaton.

In the derivation of the motional modification of the Maxwell stress tensor in vacuum, our first step is to solve the classical electromagnetic scattering by a moving mirror. In the next section, the scattered fields are computed up to first order in mirror displacement, from which follows the derivation of the vacuum radiation pressure in section 3. A general discussion of our results is presented in section 4.

2. Scattering by moving mirrors

Consider a flat perfectly reflecting mirror moving along the normal to its surface, $\hat{x}$. We assume that the electromagnetic fields E' and B' measured in the instantaneously co-moving Lorentz frame S' obey the boundary conditions

$$\hat{x} \times E' \big|_{\text{mirror}} = 0 \qquad \hat{x} \cdot B' \big|_{\text{mirror}} = 0. \tag{1}$$

As usual in electromagnetic scattering theory, we resolve the input plane waves into components corresponding to the electric field parallel (TM) or perpendicular (TE) to the plane of incidence. Since the polarization is conserved in the scattering by the flat infinite mirror, we have two independent problems to solve. We associate a scalar field with each polarization and then derive Dirichlet and Neumann boundary conditions for the scalar fields corresponding to TE and TM polarizations, respectively. We use MKS units and assume $\epsilon_0 = 1$, $c = 1$.

We analyse the scattering of TE incident waves by considering the vector potential $A^{(\mathrm{TE})}$, defined through the equations

$$E^{(\mathrm{TE})} = -\partial_t A^{(\mathrm{TE})} \qquad \text{and} \qquad B^{(\mathrm{TE})} = \nabla \times A^{(\mathrm{TE})} \tag{2}$$

and which is taken in the laboratory frame. Moreover, we take the Coulomb gauge,

$$\nabla \cdot A^{(\mathrm{TE})} = 0. \tag{3}$$

As shown in appendix A, the boundary condition for the vector potential $A^{(\mathrm{TE})}$ that follows from equations (1) and (2) is given by:

$$A^{(\mathrm{TE})}(x = \delta q(t), y, z, t) = 0 \tag{4}$$

where $\delta q(t)$ is the position of the mirror at time t.

The plane symmetry implies as well that the component of the wave vector parallel to the plane of the mirror is conserved. Hence it is convenient to work with a mixed reciprocal space, in which we take the Fourier transform of the variable $r_\parallel = (y, z)$, which corresponds to the position on the mirror's surface, while keeping the coordinate x, which is important to describe the mirror's motion. We then define the Fourier transform $F[x, k_\parallel, \omega]$ of a function $F(x, r_\parallel, t)$ as

$$F[x, k_\parallel, \omega] = \int \mathrm{d}t \int \mathrm{d}^2 r_\parallel \mathrm{e}^{\mathrm{i}\omega t} \mathrm{e}^{-\mathrm{i}k_\parallel \cdot r_\parallel} F(x, r_\parallel, t) \tag{5}$$

and use Fourier-transformed fields as in equation (5) throughout this paper. Equation (5) corresponds to a plane-wave decomposition of $F(x, r_\parallel, t)$. If $F(x, r_\parallel, t)$ is a solution of the wave equation, then $F[x, k_\parallel, \omega]$ should be of the form (cf also appendix A)

$$F[x, k_\parallel, \omega] = \exp[\pm \mathrm{i}k_x x] F[0, k_\parallel, \omega] \tag{6}$$

with

$$k_x = [(\omega + \mathrm{i}\epsilon)^2 - k_\parallel^2]^{1/2} \qquad \epsilon \to 0^+ \tag{7}$$

defined as a function in the complex plane of ω with a branch cut along the segment on the real axis between $-k_\parallel$ and $k_\parallel$ (we have set $k_\parallel = |\boldsymbol{k}_\parallel|$). Equation (6) means that given values of $\boldsymbol{k}_\parallel$ and ω correspond to a plane wave with wavevector $\boldsymbol{k} = k_x\hat{x} + \boldsymbol{k}_\parallel$. Note that imaginary values of k_x are admitted in equation (7). They correspond to evanescent waves that propagate parallel to the plane $x = 0$.

We solve the wave equation for the Dirichlet boundary condition given by equation (4). The simplest approach is to decompose the total field $\boldsymbol{A}^{(\mathrm{TE})}$ into a given incident field $\boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}$ (for instance, a plane wave, but we assume only that $\boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}$ satisfies the wave equation) plus a scattered outgoing field $\boldsymbol{A}_{\mathrm{s}}^{(\mathrm{TE})}$:

$$\boldsymbol{A}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] = \boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] + \boldsymbol{A}_{\mathrm{s}}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] \tag{8}$$

and then solve for $\boldsymbol{A}_{\mathrm{s}}^{(\mathrm{TE})}$ in terms of $\boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}$.

We derive in appendix A the scattered field up to first order in δq. we assume $x < 0$, the results for $x > 0$ following by symmetry arguments. We use $\theta(\omega)$ to represent the step function. We find

$$\boldsymbol{A}_{\mathrm{s}}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] = -\boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}[-x, \boldsymbol{k}_\parallel, \omega] + \delta\boldsymbol{A}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] \tag{9}$$

$$\delta\boldsymbol{A}^{(\mathrm{TE})}[x, \boldsymbol{k}_\parallel, \omega] = -2\mathrm{e}^{-\mathrm{i}k_x x}\int \frac{\mathrm{d}\omega'}{2\pi}\theta(k_x'^2)\delta q[\omega - \omega']\partial_x \boldsymbol{A}_{\mathrm{in}}^{(\mathrm{TE})}[0, \boldsymbol{k}_\parallel, \omega'] \tag{10}$$

with k_x and k_x' given by equation (7) as functions of ω and ω'.

In the case of TM polarization, the Lorentz boost to the co-moving frame mixes the components of the vector potential $\boldsymbol{A}$ (in the laboratory frame) parallel and perpendicular to the plane of the mirror. Then equations (1) and (2) entail a complicated boundary condition for the components of $\boldsymbol{A}$. The invariance of the free-space Maxwell equations under the duality transformation

$$\boldsymbol{E} \to \boldsymbol{B} \qquad \boldsymbol{B} \to -\boldsymbol{E}. \tag{11}$$

suggests a simpler method, which is based on the definition of a new vector potential $\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$ (taken as well in the laboratory frame) using the formulae

$$\boldsymbol{E}^{(\mathrm{TM})} = \nabla \times \boldsymbol{\mathcal{A}}^{(\mathrm{TM})} \qquad \boldsymbol{B}^{(\mathrm{TM})} = \partial_t \boldsymbol{\mathcal{A}}^{(\mathrm{TM})} \tag{12}$$

and

$$\nabla \cdot \boldsymbol{\mathcal{A}}^{(\mathrm{TM})} = 0. \tag{13}$$

The boundary condition for the potential $\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$ is derived from equations (1) and (12) in appendix A:

$$[\partial_x + \delta\dot{q}(t)\partial_t + \mathcal{O}(\delta q)^2]\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(x, \boldsymbol{r}_\parallel, t)|_{x=\delta q(t)} = 0 \tag{14}$$

where $\delta\dot{q}(t)$ is the velocity of the mirror. The first-order scattered field is found to be (for $x < 0$)

$$\boldsymbol{\mathcal{A}}_{\mathrm{s}}^{(\mathrm{TM})}[x, \boldsymbol{k}_\parallel, \omega] = \boldsymbol{\mathcal{A}}_{\mathrm{in}}^{(\mathrm{TM})}[-x, \boldsymbol{k}_\parallel, \omega] + \delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}[x, \boldsymbol{k}_\parallel, \omega] \tag{15}$$

$$\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}[x, \boldsymbol{k}_\parallel, \omega] = -2\mathrm{i}\,\frac{\mathrm{e}^{-\mathrm{i}k_x x}}{k_x}\int \frac{\mathrm{d}\omega'}{2\pi}\theta(k_x'^2)\delta q[\omega - \omega'](k_\parallel^2 - \omega\omega')\boldsymbol{\mathcal{A}}_{\mathrm{in}}^{(\mathrm{TM})}[0, \boldsymbol{k}_\parallel, \omega']. \tag{16}$$

The first terms in the RHS of (9) and (15) represent the field scattered by a mirror at rest at $x = 0$. The motional effect is represented by the terms $\delta\boldsymbol{A}^{(\mathrm{TE})}$ and $\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$. Equations (10) and (16) explicitly display the conservation of $\boldsymbol{k}_\parallel$ as well as of the polarization, in agreement with our previous discussion. Furthermore, it shows that the scattered fields propagate from the plane at $x = 0$ into the half space $x < 0$, provided that $\omega > k_\parallel$ (cf equation (6)).

Otherwise, we have evanescent waves that decay along the x-direction. Each incident spectral component at ω' generates an elastic component at $\omega = \omega'$, which is just the normally-reflected wave (first term in the RHS of equations (9) and (15)), and inelastic components (sidebands) at $\omega = \omega' + \Omega$ corresponding to each spectral component of the motion Ω. In the particular case of harmonic motion at frequency Ω (and amplitude δq_0),

$$\delta q[\omega] = \pi \delta q_0 [\delta(\omega - \Omega) + \delta(\omega + \Omega)]. \tag{17}$$

there are only two sidebands at $\omega = \omega' \pm \Omega$, with amplitudes proportional to $k'_x \delta q_0$. They propagate along different spatial directions (for they correspond to different values of k'_x), as shown in figure 1. The downshifted sideband corresponds to an evanescent wave if $\Omega > \omega' - k_\parallel$. This example is a limiting case of a more realistic situation in which the mirror is initially at rest at $x = 0$, then begins to oscillate up to a given later time, and finally stops again. We then have finite spectral bandwidths instead of the delta functions of equation (17), and the scattering of an incident plane wave generates a continuum centered around the two sidebands shown in figure 1. Actually, the important assumption used in the derivation of the scattered fields presented in appendix A is that the motion is bounded, and that the maximum displacement is much smaller than the relevant scattered wavelengths, so that the fields are nearly constant over a distance of the order of $\delta q(t)$ along the x direction.

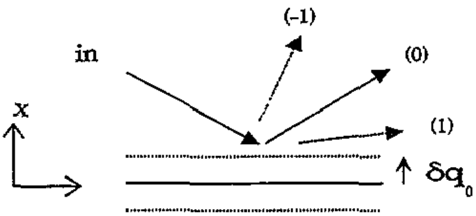

Figure 1. An input electromagnetic wave (*in*) is scattered by a plane mirror oscillating along its normal direction ($\hat{x}$). Besides the central component (0), two sidebands are generated, corresponding to upshifted (-1) and downshifted (1) frequencies. δq_0 represents the amplitude of the motion.

In the next two sections, we discuss how the vacuum fluctuations coming from infinity are modified by the presence of a moving mirror. It then turns out that the long-wavelength approximation mentioned above is related to the non-relativistic limit. Using equations (9)–(10) and (15)–(16), we compute, in the next section, the motional modification of the Maxwell stress tensor and the vacuum radiation pressure on a moving mirror.

3. Vacuum radiation pressure

In this section, we consider the motional scattering of the input vacuum fluctuations. In appendix B, we discuss the quantization of the electromagnetic fields in terms of the vector potentials $\boldsymbol{A}^{(\mathrm{TE})}$ and $\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$. According to equations (3) and (13), they are both transverse fields that satisfy the wave equation. The electric and magnetic fields are given by

$$\boldsymbol{E}(\boldsymbol{r}, t) = -\partial_t \boldsymbol{A}^{(\mathrm{TE})}(\boldsymbol{r}, t) + \nabla \times \boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(\boldsymbol{r}, t), \tag{18}$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = \nabla \times \boldsymbol{A}^{(\mathrm{TE})}(\boldsymbol{r}, t) + \partial_t \boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(\boldsymbol{r}, t). \tag{19}$$

We first consider the fluctuations coming from the half-space $x < 0$, and then use symmetry arguments to infer the effect of the fluctuations coming from the opposite side. Accordingly, we consider the free fields $A_{\rm in}^{\rm (TE)}(r,t)$ and $\mathcal{A}_{\rm in}^{\rm (TM)}(r,t)$ that propagate from $x = -\infty$ towards the mirror. Thus, their normal-mode expansions in terms of the annihilation operators $a_k^{\rm (TE)}$, $a_k^{\rm (TM)}$ and their Hermitian conjugates (corresponding to photon creation) $a_k^{\rm (TE)\dagger}$ and $a_k^{\rm (TM)\dagger}$ include only wavevectors k such that $k_x > 0$

$$\left\{ \begin{array}{c} A_{\rm in}^{\rm (TE)}(r,t) \\ \mathcal{A}_{\rm in}^{\rm (TM)}(r,t) \end{array} \right\} = \int_{(k_x>0)} \frac{{\rm d}^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2k}} \left\{ \begin{array}{c} a_k^{\rm (TE)} \\ a_k^{\rm (TM)} \end{array} \right\} {\rm e}^{{\rm i}k\cdot r}{\rm e}^{-{\rm i}kt}\hat{\epsilon}_k + {\rm HC} \tag{20}$$

where HC means the Hermitian conjugate of the preceding expression. Both polarizations are written in terms of the *same* unit vectors $\hat{\epsilon}_k$, which are perpendicular to the plane containing the vectors k and $\hat{x}$:

$$\hat{\epsilon}_k = \hat{x} \times \frac{k_\parallel}{k_\parallel}. \tag{21}$$

The operators $a_k^{(j)}$, with j representing the polarization (j = TE, TM), obey the commutation relations

$$\left[a_k^{(j)}, a_{k'}^{(j')}\right] = 0 \tag{22}$$

$$\left[a_k^{(j)}, a_{k'}^{(j')\dagger}\right] = (2\pi)^3\delta(k-k')\delta_{j,j'}. \tag{23}$$

As discussed in the previous section, it is useful to work with the mixed reciprocal space defined by equation (5), which jointly with equation (20) yield

$$A_{\rm in}^{\rm (TE)}[x, k_\parallel, \omega] = \theta(k_x{}^2)\sqrt{\frac{\hbar|\omega|}{2k_x{}^2}}\left[\theta(\omega){\rm e}^{{\rm i}k_x x}a_k^{\rm (TE)}\hat{\epsilon}_k + \theta(-\omega){\rm e}^{-{\rm i}k_x x}a_{-k}^{\rm (TE)\dagger}\hat{\epsilon}_{-k}\right] \tag{24}$$

where $k = k_x\hat{x} + k_\parallel$, with k_x taken as a function of ω and $k_\parallel$ as in equation (7). Of course, equation (20) yields an analogous expression for $\mathcal{A}_{\rm in}^{\rm (TM)}(r,t)$ in terms of the operators $a_k^{\rm (TM)}$ and their Hermitian conjugates. The correspondence between the annihilation operators and positive frequencies, on one hand, and between the creation operators and negative frequencies on the other hand, is explictly displayed in equation (24). This will be important in the discussion presented in the next section.

We now take equation (1) as operators identities for the total electric and magnetic fields. Equations (9), (10) and (15), (16) are then relations between quantum operators, thus providing representations for the operators $A_{\rm s}^{\rm (TE)}[x, k_\parallel, \omega]$ and $\mathcal{A}_{\rm s}^{\rm (TM)}[x, k_\parallel, \omega]$ in terms of the operators $a_k^{(j)}$ and $a_k^{(j)\dagger}$, which correspond to the normal-mode decomposition of the input field.

We assume that the mirror is initially at rest at $x = 0$ ($t \to -\infty$). Before the mirror starts its motion, the scattered field is just the normally-reflected field. Its normal mode decomposition is written exactly as in equation (20)—except for the sign corresponding to the propagation along the x-direction and for a phase factor in the case of TE polarization—in terms of the same operators $a_k^{(j)}$. Accordingly, the vacuum state $|0\rangle_{\rm in}$ with respect to the operators $a_k^{(j)}$ corresponds to the zero-point fluctuations at $t \to -\infty$, which are modified on account of the mirror's motion. Such motional effect is described through the modification of the field operators (the Heisenberg picture, but there is no Hamiltonian formalism, as shown in [10]) according to equations (10) and (16).

The connection with mechanical effects is provided by the (mean) Maxwell stress tensor, written in terms of suitably defined symmetrical correlation functions, taken over the vacuum input state $|0\rangle_{\rm in}$. We show that the motional modification of the fields entails a mean radiation pressure δp on the mirror, which is given by

$$\delta p = \langle 0|_{\rm in} T_{xx}(x=0^+) - T_{xx}(x=0^-)|0\rangle_{\rm in} + \mathcal{O}(\delta q^2) \tag{25}$$

with the xx component of the Maxwell stress tensor given by

$$T_{xx}(x, \boldsymbol{r}_\parallel, t) = \frac{1}{2}(E_x^2 + B_x^2 - E_\parallel^2 - B_\parallel^2). \tag{26}$$

From now on we use the shortening $\langle\ldots\rangle$ to denote the average over the state $|0\rangle_{\rm in}$. We decompose the electric and magnetic fields as in equations (18) and (19). We then take the Fourier transform of equation (26) and find

$$\langle T_{xx}[x, \boldsymbol{k}_\parallel, \omega]\rangle = \frac{1}{4}\int \frac{{\rm d}^2k'_\parallel}{(2\pi)^2}\int\frac{{\rm d}\omega'}{2\pi}\{[k'^2_\parallel + \omega'(\omega-\omega')] - \partial_{x_1}\partial_{x_2}\}$$
$$\times\sigma[x_1, \boldsymbol{k}'_\parallel, \omega'; x_2, -\boldsymbol{k}'_\parallel, \omega-\omega']\,\big|_{x_1=x_2=x} \tag{27}$$

where $\sigma[x_1, \boldsymbol{k}_{1\parallel}, \omega_1; x_2, \boldsymbol{k}_{2\parallel}, \omega_2] \equiv \sigma[1;2]$ is the sum of the symmetrical correlation functions

$$\sigma^{\rm (TE)}[1;2] = \langle\{A^{\rm (TE)}[x_1, \boldsymbol{k}_{1\parallel}, \omega_1], A^{\rm (TE)}[x_2, \boldsymbol{k}_{2\parallel}, \omega_2]\}\rangle \tag{28}$$

and

$$\sigma^{\rm (TM)}[1;2] = \langle\{\mathcal{A}^{\rm (TM)}[x_1, \boldsymbol{k}_{1\parallel}, \omega_1], \mathcal{A}^{\rm (TM)}[x_2, \boldsymbol{k}_{2\parallel}, \omega_2]\}\rangle \tag{29}$$

with $\{\,,\}$ designating the anticommutator.

The symmetrical correlation function of the input field $A_{\rm in}^{\rm (TE)}$ as well as of $\mathcal{A}_{\rm in}^{\rm (TM)}$ is easily derived from equation (24),

$$\sigma_{\rm in}^{\rm (TE)}[1;2] = \langle\{A_{\rm in}^{\rm (TE)}[x_1, \boldsymbol{k}_{1\parallel}, \omega_1]A_{\rm in}^{\rm (TE)}[x_2, \boldsymbol{k}_{2\parallel}, \omega_2]\}\rangle,$$
$$\sigma_{\rm in}^{\rm (TE)}[1;2] = 4\pi^3\hbar\theta(k_{1x}^2)\frac{{\rm e}^{{\rm i}k_{1x}(x_1-x_2)}}{|k_{1x}|}\delta(\omega_1+\omega_2)\delta(\boldsymbol{k}_{1\parallel}+\boldsymbol{k}_{2\parallel}) \tag{30}$$

where $k_{1x} = [(\omega_1 + {\rm i}\epsilon)^2 - k_{1\parallel}^2]^{1/2}$; and of course,

$$\sigma_{\rm in}^{\rm (TM)}[1;2] = \sigma_{\rm in}^{\rm (TE)}[1;2]. \tag{31}$$

Since the average radiation pressure on a standing mirror vanishes, only the motional modification of the stress tensor T_{xx} contributes to equation (25). We set

$$\langle T_{xx}\rangle = \langle T_{xx}^0\rangle + \langle\delta T_{xx}\rangle \tag{32}$$

where $\langle T_{xx}^0\rangle$ is the stress tensor for a mirror at rest at $x=0$, and $\langle\delta T_{xx}\rangle$ its motional correction to first order in δq. The latter is obtained by replacing the first-order correction to the correlation function σ into equation (27). Accordingly, we write

$$\sigma^{(j)} = \sigma_0^{(j)} + \delta\sigma^{(j)} \qquad j = {\rm TE, TM} \tag{33}$$

where $\sigma_0^{(j)}$ corresponds to the correlation functions when the mirror is at rest at $x=0$, and $\delta\sigma^{(j)}$ its first-order correction, corresponding to the motional effect.

The *simplest representation is to write* $\delta\sigma^{(j)}[1;2]$ *as the symmetrical part of a function* $\delta c^{(j)}[1;2]$:

$$\delta\sigma^{(j)}[1;2] = \delta c^{(j)}[1;2] + \delta c^{(j)}[2;1]\,. \tag{34}$$

From equations (10) and (30), we derive the following result for $j = {\rm TE}$:

$$\delta c^{\rm (TE)}[1;2] = -8\pi^2\hbar\theta(k_{1x}^2)\sin(|k_{1x}|x_1){\rm e}^{-{\rm i}k_{2x}x_2}\delta q[\omega_1+\omega_2]\delta(\boldsymbol{k}_{1\parallel}+\boldsymbol{k}_{2\parallel}). \tag{35}$$

Instead of equation (10), we use equation (16) in order to obtain $\delta c^{(\mathrm{TM})}[1;2]$:

$$\delta c^{(\mathrm{TM})}[1;2] = -8\mathrm{i}\pi^2\hbar\theta(k_{1x}^2)\frac{(\omega_1\omega_2 + k_{1\|}^2)}{|k_{1x}|k_{2x}}\cos(k_{1x}x_1)\mathrm{e}^{-\mathrm{i}k_{2x}x_2} \times\delta q[\omega_1+\omega_2]\delta(\boldsymbol{k}_{1\|}+\boldsymbol{k}_{2\|}). \tag{36}$$

The contribution of TE-polarized waves to the motional modification of the stress tensor is then found by replacing equations (34) and (35) into equation (27) and evaluating the resulting integral:

$$\langle\delta T_{xx}^{(\mathrm{TE})}[x,\boldsymbol{k}_\|,\omega]\rangle = \hbar\delta q[\omega]\delta(\boldsymbol{k}_\|)\frac{\mathrm{e}^{-\mathrm{i}\omega x}}{12}\left(-\frac{\omega^2}{x^3} - \mathrm{i}\frac{\omega^3}{x^2} + \frac{2}{5}\frac{\omega^4}{x}\right) \tag{37}$$

for $x < 0$. The result for $x > 0$ is found by replacing x by $-x$ and changing the sign in equation (37). Not surprisingly, we find Ford's and Vilenkin's result [4] for a scalar field obeying Dirichlet boundary condition on a flat moving mirror. For us, this is an auxiliary result in the derivation of stress tensor for the electromagnetic field. We have to add to the RHS of (37) the contribution corresponding to TM-polarized waves, which is obtained in a similar way, by substituting equation (36) into equation (27):

$$\langle\delta T_{xx}^{(\mathrm{TM})}[x,\boldsymbol{k}_\|,\omega]\rangle = \hbar\delta q[\omega]\delta(\boldsymbol{k}_\|)\frac{\mathrm{e}^{-\mathrm{i}\omega x}}{12}\left(\frac{\omega^2}{x^3} + \mathrm{i}\frac{\omega^3}{x^2} + \frac{2}{5}\frac{\omega^4}{x}\right). \tag{38}$$

Adding the RHS of equations (37) and (38) leads to the motional modification of the electromagnetic stress tensor, which is written in the time domain as

$$\langle\delta T_{xx}(x,t)\rangle = \frac{\hbar}{60\pi^2 x}\delta q^{(4)}(t-|x|) \tag{39}$$

where $\delta q^{(n)}(t)$ represents the nth time derivative of the mirror's displacement $\delta q(t)$. Finally, we derive the radiation pressure by evaluating equation (39) at $x = \delta x \to 0$ and using equation (25):

$$\delta p(t) = \frac{\hbar}{30\pi^2\delta x}\delta q^{(4)}(t) - \frac{\hbar}{30\pi^2}\delta q^{(5)}(t). \tag{40}$$

4. Discussion

In this section we discuss the meaning of equation (40), which is the central result of this paper. First, we show that this result is in full agreement with the fluctuation–dissipation theorem, which is more easily stated in the spectral domain. The motional radiation pressure given by equation (40) is then written in terms of a susceptibility function $\chi[\omega]$ and of the surface of the mirror A:

$$\delta p[\omega] = \chi[\omega]\delta q[\omega]/A \tag{41}$$

$$\chi[\omega] = \frac{\hbar A}{30\pi^2}\left(\frac{\omega^4}{\delta x} + \mathrm{i}\omega^5\right). \tag{42}$$

The imaginary part of $\chi[\omega]$ represents the dissipative part of the motional force. As we show below, $\mathrm{Im}\,\chi[\omega]$ is related to the spectrum of fluctuations of the force exerted by vacuum on the mirror at rest, defined as

$$C_{FF}[\omega] = \int \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}(\langle F(t)F(0)\rangle - \langle F\rangle\mathrm{e}^2). \tag{43}$$

The result for $C_{FF}[\omega]$ found in [7, 8] is

$$C_{FF}[\omega] = \frac{\hbar^2 A}{15\pi^2 c^4}\theta(\omega)\omega^5. \tag{44}$$

It follows from equations (42) and (44) that the dissipative component of susceptibility and the spectrum of fluctuations are related by

$$\mathrm{Im}\chi[\omega] = \frac{1}{2\hbar}(C_{FF}[\omega] - C_{FF}[-\omega]) \tag{45}$$

in agreement with the fluctuation–dissipation theorem [3].

Far more intriguing is the dispersive component appearing as well in equation (42), which diverges when δx vanishes. Such anomaly also occurs for the three-dimensional Dirichlet scalar field considered by Ford and Vilenkin [4] (cf equation (37)). The need for a re-examination of this result was recently emphasized by Barton (see the final remarks in [7]). As discussed below, the physical origin of both dissipative and dispersive components may be clearly identified within our approach, allowing us to check the validity of the assumptions that led to equation (40).

In the linear approximation, the total radiation pressure is the superposition of the pressure corresponding to each spectral component of the motion, which may be treated separately (cf equation (41)). On the other hand, the input fields may also be decomposed *into their plane-wave components, which are specified by given values of frequency* and parallel wavevector, ω_{in} and $\boldsymbol{k}_{\|}$. The susceptibility $\chi[\omega]$ specifies, according to equation (41), the combined effect of all the plane-wave components scattered by the motional Fourier component at ω. The elementary processes in which a given motional frequency ω scatters a given plane-wave component $(\omega_{\mathrm{in}}, \boldsymbol{k}_{\|})$ was analysed in detail in section 2, where we showed that it generates a sideband at $\omega_{\mathrm{s}} = \omega_{\mathrm{in}} + \omega$ with the same value of $\boldsymbol{k}_{\|}$. We may derive a representation in which $\chi[\omega]$ explicitly appears as a superposition of all the relevant plane-wave components by taking the limit $x \to 0$ from the beginning in equations (27), (35) and (36),

$$\chi[\omega] = 2\mathrm{i}A\hbar \int \frac{\mathrm{d}^2 k_{\|}}{(2\pi)^2} \int \frac{\mathrm{d}\omega_{\mathrm{in}}}{2\pi} \theta({k_{\mathrm{in}}}_x^2) \left[\frac{(k_{\|}^2 - \omega_{\mathrm{in}}\omega_{\mathrm{s}})^2}{|k_{\mathrm{in}x}| k_{\mathrm{s}x}} + |k_{\mathrm{in}x}| k_{\mathrm{s}x} \right] \tag{46}$$

where $k_{\mathrm{in}x} = (\omega_{\mathrm{in}}^2 - k_{\|}^2)^{1/2}$ and $k_{\mathrm{s}x} = [(\omega_{\mathrm{s}} + \mathrm{i}\epsilon)^2 - k_{\|}^2]^{1/2}$ represent the x component of the input and output wavevectors, respectively .

The region of integration in equation (46) represents the entire set of Fourier components that correspond to propagating waves. It is divided in four subsets, denoted R_1 to R_4, that are shown in figure 2 (where ω is taken to be positive). Input evanescent waves, which are *represented by the grey region in the figure, do not contribute to* $\chi[\omega]$ *because there are no* evanescent-wave solutions for a perfect mirror at rest and hence no first-order correction to the correlation function σ.

We may represent graphically the generation of a sideband from a given plane-wave component $(\omega_{\mathrm{in}}, \boldsymbol{k}_{\|})$ in figure 2 as a horizontal shift, by an amount of ω, from the point that represents $(\omega_{\mathrm{in}}, \boldsymbol{k}_{\|})$ in the diagram. Thus the region R_4 represents input propagating waves that scatter into evanescent waves: $|\omega_{\mathrm{s}}| < k_{\|}$, whereas R_1 to R_3 contain those that give rise to propagating waves, R_3 being the subset corresponding to the negative input frequencies ω_{in} that generate propagating waves at the positive part of the spectrum. Equation (46) immediately shows that the imaginary (dissipative) and real (dispersive) parts of $\chi[\omega]$ come from R_3 and R_4, respectively, whereas the integrals over R_1 and R_2 cancel each other. In fact, performing the integral over R_3 leads to the dissipative pressure already found in equation (40). Thus, equation (46) relates the dissipative pressure to the map of negative into positive frequencies (and vice versa, because the motional spectrum necessarily contains both positive and negative frequencies since $\delta q(t)$ is real). Furthermore, the negative input frequencies which are scattered into positive frequencies corresponding to evanescent

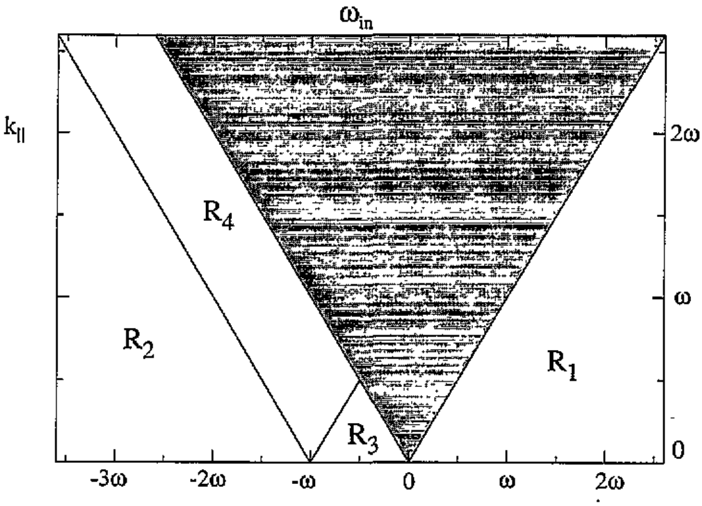

Figure 2. Diagram representing input waves scattered by a plane mirror. Each point corresponds to a given input frequency $\omega_{\rm in}$ and transverse wavevector $k_\parallel$. The grey region contains the input evanescent waves ($\omega_{\rm in} < k_\parallel$), which do not contribute to the motional effect. Regions R_1 to R_4, contain input travelling waves, R_3 and R_4 providing the dissipative and dispersive components of the radiation pressure at frequency ω, respectively. The contributions from R_1 and R_2 cancel each other.

waves (the low-frequency part of R_4 in figure 2) contribute to the dispersive and not to the dissipative component. As shown by equation (24), positive and negative input frequencies correspond to annihilation and creation operators. Therefore, our approach indicates that the dissipative motional force is related to the creation of travelling-wave photons, in agreement with its interpretation as a radiation-reaction force. However, we cannot compute the radiated energy within the linear approximation. In fact, we may follow [4] and assume the radiated energy rate to be equal to the power dissipated by the motional force. The energy radiated is then given by

$$E = -\int_{-\infty}^{\infty} \mathrm{d}t\, F(t)\, \delta\dot{q}(t). \tag{47}$$

We then find from equations (40) and (47)

$$E = \frac{\hbar}{30\pi^2}\int_{-\infty}^{\infty} \mathrm{d}t\, (\delta q^{(3)}(t))^2. \tag{48}$$

As in the classical electron theory, a linear radiation reaction force implies a second-order rate of energy radiation.

We may now discuss how the linear approximation is related to the non-relativistic limit. If

$$\int \mathrm{d}\omega\, |\omega\, \delta q\, [\omega]| \ll 1 \tag{49}$$

then $|\delta\dot{q}(t)| \ll 1$. Since the input modes that contribute to the dissipative force belong to R_3, they satisfy $\omega_{\rm in} \leqslant \omega$. Thus, the long-wavelength approximation,

$$|\omega_{\rm in}\, \delta q(t)| \ll 1$$

which is basic for the linear expansion, follows from equation (49), which is slightly stronger than the non-relativistic assumption (ruling out unbounded non-relativistic motions).

The consistency of the assumption of perfect reflectivity may be checked in a similar way. Of course, any real mirror has a finite transparency frequency ω_c, so that the waves at frequencies $\omega \gtrsim \omega_c$ are not reflected. If the motional spectral components are much smaller than ω_c, then all the plane wave components that contribute to the dissipative force are perfectly reflected, and thus our model is realistic in this case. Therefore, the susceptibility function $\chi[\omega]$ given by equation (42) should be regarded as a low-frequency approximation of the susceptibility function of a real mirror. This explains why dispersion relations are useless for such a model, since they involve the whole spectrum of frequencies.

In other words, the results obtained in this paper for the dissipative force apply for a mirror that moves slowly (i.e. besides being small, the mirror's velocity varies smoothly in time) on the time scale of its internal degrees of freedom (which is of the order of $1/\omega_c$). Such internal variables (currents and charges near the mirror's internal surface) will just follow the field variations in this case, and may be completely ignored, as is usual in the adiabatic limit.

However, the computation of the dispersive force is clearly inconsistent with our assumptions, because $\mathrm{Re}\,\chi[\omega]$ results from the integration over the unbounded region R_4 in figure 2. Accordingly, we understand why the model leads to unphysical results in the case of a three-dimensional space, for both electromagnetic and scalar fields (cf [4]). On the other hand, no divergence occurs for one-dimensional models [2, 4], since the scattering does not generate evanescent waves in this case (incidentally, this limiting case corresponds to the scattering of waves with $k_\parallel = 0$, so that the one-dimensional linear model may be considered from the results presented here as well).

As a final remark, we note that the derivation of the stress tensor $\langle \delta T_{xx}(x,t)\rangle$ given by equation (39) is self-consistent when $|x| \gg |\delta q(t)|$, for the main contribution in the integral of equation (27) comes from input waves of frequencies $\omega_{\rm in} \lesssim 1/|x|$, hence justifying the linear expansion. In other words, high-frequency evanescent waves do not contribute considerably at points far from the mirror's surface, thus justifying our assumptions in this case.

In conclusion, we have derived the electromagnetic fields scattered by a perfectly-reflecting moving mirror in the long-wavelength limit, allowing us to compute the modification of the Maxwell stress tensor induced by a non-relativistic bounded motion in vacuum. The contribution of high-frequency plane-wave field components which are scattered into evanescent waves leads to a divergent dispersive force. On the other hand, the dissipative force is related to the motional creation of travelling-wave photons. Our result agrees with the fluctuation–dissipation theorem of linear response theory.

Acknowledgments

The author thanks C Eberlein, M-T Jaekel and S Reynaud for discussions and remarks. This work was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico.

Appendix A. Derivation of the scattered fields

We derive the fields scattered by a moving mirror up to first order in the mirror's displacement δq. Our method is based on the decomposition of the electromagnetic fields into TE and TM components followed by a careful choice of the vector potentials used in the representation of each component.

We first decompose the Fourier transformed electric field (defined as in equation (5)) as

$$E[x, k_\parallel, \omega] = E^{(\mathrm{TE})}[x, k_\parallel, \omega] + E^{(\mathrm{TM})}[x, k_\parallel, \omega] \tag{A1}$$

where $E^{(\mathrm{TE})}[x, k_\parallel, \omega]$ is the component along the direction perpendicular to the plane containing $\hat{x}$ and $k_\parallel$ (plane of incidence). Then we define $E^{(\mathrm{TE})}(r, t)$ as the inverse Fourier transform of $E^{(\mathrm{TE})}[x, k_\parallel, \omega]$, and proceed likewise for $E^{(\mathrm{TM})}$. Equation (A1) yields

$$E(r, t) = E^{(\mathrm{TE})}(r, t) + E^{(\mathrm{TM})}(r, t). \tag{A2}$$

We have an analogous decomposition for the magnetic field:

$$B(r, t) = B^{(\mathrm{TE})}(r, t) + B^{(\mathrm{TM})}(r, t). \tag{A3}$$

The decomposition entails

$$E^{(\mathrm{TE})}(r, t) \cdot \hat{x} = 0 \qquad B^{(\mathrm{TM})}(r, t) \cdot \hat{x} = 0. \tag{A4}$$

The simplest way to implement the Lorentz boost along the x-direction is to represent the TE and TM field components in equations (A2) and (A3) by the vector potentials $A^{(\mathrm{TE})}(r, t)$ and $\mathcal{A}^{(\mathrm{TM})}(r, t)$ as in equations (2) and (12)—cf equations (18) and (19), which jointly with equation (A4) entail $A^{(\mathrm{TE})}(r, t) \cdot \hat{x} = \mathcal{A}^{(\mathrm{TM})}(r, t) \cdot \hat{x} = 0$. Therefore, the vector potentials $A'^{(\mathrm{TE})}(r', t')$ and $\mathcal{A}'^{(\mathrm{TM})}(r', t')$ measured in the Lorentz instantaneously co-moving frame obey the relations

$$A'^{(\mathrm{TE})}(r', t') = A^{(\mathrm{TE})}(r, t) \qquad \mathcal{A}'^{(\mathrm{TM})}(r', t') = \mathcal{A}^{(\mathrm{TM})}(r, t). \tag{A5}$$

Equation (A5) yields the following expressions for the electric field measured by the co-moving observer on the surface of the mirror:

$$E'^{(\mathrm{TE})}\big|_{\mathrm{mirror}} = -\big(\partial_t + \delta\dot{q}(t)\partial_x + \mathcal{O}(\delta q^2)\big)A^{(\mathrm{TE})}(\delta q(t), r_\parallel, t) \tag{A6}$$

$$\hat{x} \times E'^{(\mathrm{TM})}\big|_{\mathrm{mirror}} = -\big(\partial_x + \delta\dot{q}(t)\partial_t + \mathcal{O}(\delta q^2)\big)\mathcal{A}^{(\mathrm{TM})}(\delta q(t), r_\parallel, t). \tag{A7}$$

Combining equations (1) and (A7) immediately yields equation (14). On the other hand, equation (A6) relates $E'^{(\mathrm{TE})}\big|_{\mathrm{mirror}}$ to the total time derivative of $A^{(\mathrm{TE})}(\delta q(t), r_\parallel, t)$. Equation (1) then entails a constant value for $A^{(\mathrm{TE})}(\delta q(t), r_\parallel, t)$, which is taken to be zero without loss of generality, in accordance with (4).

The solution of the boundary conditions given by equations (4) and (14) is as follows. We first consider the Dirichlet boundary condition given by equation (4). We assume that the fields are nearly constant over a distance of the order of $\delta q(t)$. We use equation (8) and then expand equation (4) around $x = 0$ up to first order:

$$A_s^{(\mathrm{TE})}(0, r_\parallel, t) + \delta q(t)\partial_x A_s^{(\mathrm{TE})}(0, r_\parallel, t) = -\big[A_{\mathrm{in}}^{(\mathrm{TE})}(0, r_\parallel, t) + \delta q(t)\partial_x A_{\mathrm{in}}^{(\mathrm{TE})}(0, r_\parallel, t)\big]. \tag{A8}$$

For a mirror at rest, the scattered field is given by

$$A_s^{(\mathrm{TE})}(x, r_\parallel, t) = -A_{\mathrm{in}}^{(\mathrm{TE})}(-x, r_\parallel, t), \tag{A9}$$

which corresponds to the wave reflected by a perfect mirror at rest at $x = 0$. The Fourier transform of the field given by equation (A9) is indeed the first term in the RHS of equation (9). It then follows that

$$\partial_x A_s^{(\mathrm{TE})}[0, k_\parallel, \omega'] = \mathrm{sgn}(\omega'^2 - k_\parallel^2)\partial_x A_{\mathrm{in}}^{(\mathrm{TE})}[0, k_\parallel, \omega'] + \mathcal{O}(\delta q), \tag{A10}$$

where $\mathrm{sgn}(x) = \theta(x) - \theta(-x)$ is the sign function.

We consider the effect of the mirror motion to be a small perturbation. Accordingly, we replace the partial derivative of the scattered field in the RHS of (A8)—which is already of first order—by using the zeroth-order result given by (A10), in order to obtain a boundary condition for the scattered field which is valid up to first order in $\delta q[\omega]$:

$$A_{\rm s}^{(\rm TE)'}[0, \boldsymbol{k}_\parallel, \omega] = -A_{\rm in}^{(\rm TE)}[0, \boldsymbol{k}_\parallel, \omega] \\ -2\int \frac{\mathrm{d}\omega'}{2\pi}\theta(\omega'^2 - k_\parallel^2)\delta q[\omega-\omega']\partial_x A_{\rm in}^{(\rm TE)}[0, \boldsymbol{k}_\parallel, \omega']. \tag{A11}$$

We have thus transformed the homogeneous Dirichlet condition on a moving boundary given by equation (4) into the inhomogeneous condition on a plane *at rest* at $x = 0$ given by equation (A11), which may be more easily solved.

The scattered field at $x < 0$ is a solution of the wave equation written in the mixed reciprocal space:

$$(\partial_x^2 + \omega^2 - k_\parallel^2)A_{\rm s}^{(\rm TE)}[x, \boldsymbol{k}_\parallel, \omega] \tag{A12}$$

which propagates (or is damped) from the plane $x = 0$ into the half-space $x < 0$:

$$A_{\rm s}^{(\rm TE)}[x, \boldsymbol{k}_\parallel, \omega] = \exp(-\mathrm{i}k_x x)A_{\rm s}^{(\rm TE)}[0, \boldsymbol{k}_\parallel, \omega] \tag{A13}$$

with k_x defined as in equation (7). For a standing mirror, we thus have $A_{\rm s}^{(\rm TE)}[0, \boldsymbol{k}_\parallel, \omega] = -A_{\rm in}^{(\rm TE)}[0, \boldsymbol{k}_\parallel, \omega]$, and then equation (A13) yields the result for the normally-reflected field, equation (A9). The first-order motional correction, equation (10), follows from (A11) and (A13).

The solution of the boundary condition (14), corresponding to TM-polarized waves, is obtained by a similar method. We first transform (14) into an inhomogenous Neumann boundary condition by using the zeroth-order result

$$\mathcal{A}_{\rm s}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega] = \mathrm{sgn}(\omega^2 - k_\parallel^2)\mathcal{A}_{\rm in}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega] + \mathcal{O}(\delta q). \tag{A14}$$

Equations (14) and (A14) then yield:

$$\partial_x\mathcal{A}_{\rm s}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega] = -\partial_x\mathcal{A}_{\rm in}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega] - 2\int \frac{\mathrm{d}\omega'}{2\pi}\theta(k_x'^2)\delta q[\omega-\omega'] \\ \times(k_\parallel^2 - \omega\omega')\mathcal{A}_{\rm in}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega']. \tag{A15}$$

The field $\mathcal{A}_{\rm s}^{(\rm TM)}(\boldsymbol{r}, t)$ is also a solution of the wave equation (A12) that propagates into the half-space $x < 0$ according to equation (A13). Hence it may be written as:

$$\mathcal{A}_{\rm s}^{(\rm TM)}[x, \boldsymbol{k}_\parallel, \omega] = \frac{\mathrm{i}\exp(-\mathrm{i}k_x x)}{k_x}\partial_x\mathcal{A}_{\rm s}^{(\rm TM)}[0, \boldsymbol{k}_\parallel, \omega]. \tag{A16}$$

Finally, equations (15) and (16) follow from equations (A15) and (A16).

Equation (16) predicts a divergent motional contribution when the output direction is parallel to the plane of the mirror ($k_x = 0$). This effect is similar to the Woods anomalies of light scattering by diffraction gratings [13]. One may then be sceptical about equation (A15) since it was derived by assuming the motional effect to be small. In fact, a more general result may be obtained if the output field is not replaced by the RHS of equation (A14) in the first-order term of the expansion of equation (14). We then find a non-linear result which on one hand is regular at $\omega = k_\parallel$ and on the other hand agrees with equations (15) and (16) at frequencies corresponding to output fields far from the grazing direction [14]. Although the linear approximation breaks down for output fields along the grazing direction, the stress tensor in vacuum is not modified, essentially because the non-linear correction occurs over a very small region of the spectrum.

Appendix B. Canonical quantization with the potentials $A^{(\mathrm{TE})}$ and $\mathcal{A}^{(\mathrm{TM})}$

In this appendix, we develop the canonical quantization of the electromagnetic fields in terms of the vector potentials $A^{(\mathrm{TE})}$ and $\mathcal{A}^{(\mathrm{TM})}$ in free space. Following the method of [15], we work with the reciprocal space, and then define the Fourier transformed field $A_t^{(\mathrm{TE})}[k]$ as

$$A_t^{(\mathrm{TE})}[k] = \int \mathrm{d}^3r\, \mathrm{e}^{-\mathrm{i}k\cdot r} A^{(\mathrm{TE})}(r,t) \tag{B1}$$

and likewise for the field $\mathcal{A}_t^{(\mathrm{TM})}[k]$. Note that the definition in equation (B1) is different from our previous one, given by equation (5).

The standard Lagrangian of electrodynamics is

$$L_{\mathrm{st}} = \frac{1}{2}\int \mathrm{d}^3r \left[E^2(r,t) - B^2(r,t)\right]. \tag{B2}$$

We decompose the electric and magnetic fields $E_t[k]$ and $B_t[k]$ into their TE and TM components as in equations (18) and (19) to obtain the following representation:

$$L_{\mathrm{st}} = \int_{half} \frac{\mathrm{d}^3k}{(2\pi)^3}\, \mathcal{L}_{\mathrm{st}}(A_t^{(\mathrm{TE})}[k], \mathcal{A}_t^{(\mathrm{TM})}[k], \dot{A}_t^{(\mathrm{TE})}[k], \dot{\mathcal{A}}_t^{(\mathrm{TM})}[k]) \tag{B3}$$

where the Lagrangian density in reciprocal space $\mathcal{L}_{\mathrm{st}}$ is given by

$$\mathcal{L}_{\mathrm{st}} = |\dot{A}_t^{(\mathrm{TE})}[k]|^2 - k^2|A_t^{(\mathrm{TE})}[k]|^2 - \left(|\dot{\mathcal{A}}_t^{(\mathrm{TM})}[k]|^2 - k^2|\mathcal{A}_t^{(\mathrm{TM})}[k]|^2\right). \tag{B4}$$

The integral in equation (B3) includes only one half of the volume of the reciprocal space, so that the potentials $\mathcal{A}_t^{(\mathrm{TM})}[k]$ and $A_t^{(\mathrm{TE})}[k]$ may be considered as independent dynamical variables, the fields at the other half volume being determined by the relation $A_t^{(\mathrm{TE})}[-k] = A_t^{(\mathrm{TE})*}[k]$, and likewise for $\mathcal{A}_t^{(\mathrm{TM})}[k]$, which follows from the reality of $A^{(\mathrm{TE})}(r,t)$ and $\mathcal{A}^{(\mathrm{TM})}(r,t)$. They represent the components of the corresponding vector fields along the polarization unit vector $\hat{\epsilon}_k$ given by equation (21).

The terms corresponding to the TM-polarized fields appear with a minus sign in equation (B4). This is a consequence of the odd parity of L_{st} under the duality transformation (cf equation (B2)). We may easily derive the following Hamiltonian from equations (B3) and (B4):

$$H = \int_{\mathrm{half}} \frac{\mathrm{d}^3k}{(2\pi)^3} \left[|\dot{A}_t^{(\mathrm{TE})}[k]|^2 + k^2|A_t^{(\mathrm{TE})}[k]|^2 - \left(|\dot{\mathcal{A}}_t^{(\mathrm{TM})}[k]|^2 + k^2|\mathcal{A}_t^{(\mathrm{TM})}[k]|^2\right)\right] \tag{B5}$$

which does not correspond to the total energy. We may remedy such a problem by taking a Lagrangian density invariant under the duality transformation:

$$\mathcal{L}_{\mathrm{dual}} = \left|\dot{A}_t^{(\mathrm{TE})}[k]\right|^2 - k^2|A_t^{(\mathrm{TE})}[k]|^2 + |\dot{\mathcal{A}}_t^{(\mathrm{TM})}[k]|^2 - k^2|\mathcal{A}_t^{(\mathrm{TM})}[k]|^2. \tag{B6}$$

Of course, Lagrange's equations that follow from equation (B6) are identical to those following from the standard Lagrangian given by (B4). However, the former leads to a Hamiltonian which corresponds to the total field energy

$$\frac{1}{2}\int \mathrm{d}^3r \left[E^2(r,t) + B^2(r,t)\right].$$

The canonical quantization from $\mathcal{L}_{\mathrm{dual}}$ follows in the usual way (cf [15]), leading to equations (18)–(23).

References

[1] Fulling S A and Davies P C W 1976 *Proc. R. Soc.* A **348** 393
[2] Jaekel M T and Reynaud S 1992 *Quantum Optics* **4** 39
[3] Kubo R 1966 *Rep. Prog. Phys.* **29** 255, especially section 5
[4] Ford L H and Vilenkin A 1982 *Phys. Rev.* D **10** 2569
[5] Barton G 1991 *J. Phys. A: Math. Gen.* **24** 991; Barton G 1991 *J. Phys. A: Math. Gen.* **24** 5533
[6] Braginsky V B and Khalili F Ya 1991 *Phys. Lett.* **161A** 197
[7] Barton G 1993 *New aspects of the Casimir effect: fluctuations and radiative reaction*, submitted to *Cavity Quantum Electrodynamics (Supplement: Advances in Atomic, Molecular and Optical Physics)* ed P Berman (New York: Academic).
[8] Maia Neto P A and Reynaud S 1993 *Phys. Rev.* A **47** 1639
[9] Eberlein C 1993 *J. Phys. France* **I3** 2151
[10] Moore G T 1970 *J. Math. Phys.* **11** 2679
[11] Barton G and Eberlein C 1993 *Ann. Phys., NY* **227** 222
[12] At first sight, the assumption of perfect reflectivity, which implies an instantaneous response to the field variations, may be exclusively blamed for this anomaly. However, even an improved causal model for the mirror has led to a response function containing an instantaneous component, cf Jaekel M T and Reynaud S 1992 *Phys. Lett.* **167A** 227
[13] Rayleigh J W S 1907 *Proc. R. Soc.* A **79** 399
[14] Maia Neto P A 1993 *Optics Commun.* to be published
[15] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1987 *Photons et Atomes* ch 2 (Paris: InterEditions)